McDONALD

Statically Indeterminate Stresses

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STATICALLY INDETERMINATE STRESSES

BY

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THESIS

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THIS IS TO CERTIFY THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

LEWIS McDONALD

ENTITLED STATICALLY INDETERMINATE STRESSES

IS APPROVED BY ME AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE

DEGREE OF Bachelor of Science in Civil Engineering

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OUTLINE.

I. INTRODUCTION.

It is customary in most cases of Engineering design, to avoid using members whose stresses are statically indeterminate. There are, however, cases of design in which it is not possible, or at least not practicable, to do this, as in the case of cross-frames of plate girders or the central panels of riveted trusses. In such cases the use of two diagonal members makes a redundant system and hence the stresses in its members cannot be determined by the principles of statics alone.

In view of the fact that in such cases it is customary to make some assumption in regard to the stresses in the redundant members and then solve by the principles of statics it was thought well to investigate some such systems as actually built, and compare the actual results with the values for which the members were designed.

By this means it is possible to make comparisons and form some estimate as to how great an error is likely to occur in using the ordinary assumptions.

The method of least work has been used, since it is the only method for arriving at the result.

II. THEORY.

1. Principle of Least Work.

A Redundant system is one which contains more members than necessary to make the framework a rigid structure. While, in general, a redundant system can easily be detected and the number of redundant members determined, there are cases of complicated design in which this cannot be done.

To investigate such a system let m be the number of members and n the number of points. Then to form a rigid framework the first triangle contains three sides and three points and each triangle added thereto adds two sides and one point. Hence, we have

$$m-3 = 2(n-3)$$

or

$$m = 2n-3$$

as the relation between the number of sides and points to form a rigid structure. A value of m greater than is given by the equation for any system indicates

that the system contains redundant members and the number of extra members is given by subtracting the computed value of m from the actual value. Thus if a framework contains eight members and the equation gives a value of m as six the system must contain two redundant members.

The determination of the stresses in the members of a redundant system is not possible by the ordinary methods of statics, since there are more unknown stresses about a point than there are conditions of equilibrium. It is, therefore, necessary to introduce some other condition, and this is done by determining the relation between the distortion of the necessary members and the redundant members.

The general formula for the deflection of a joint in a rigid framework due to any loading is:

$$\Delta = \sum \frac{SUL}{AE}$$

where Δ = the movement of the joint,

S = the stress in any member due to a given loading,

U = the stress in the member due to a load of one pound acting at the joint in a direction opposite to its movement,

L = the length of the member,

A = the cross-section area of the member, and

E = the modulus of elasticity of the material.

2. Application of Least Work to Redundant Members.

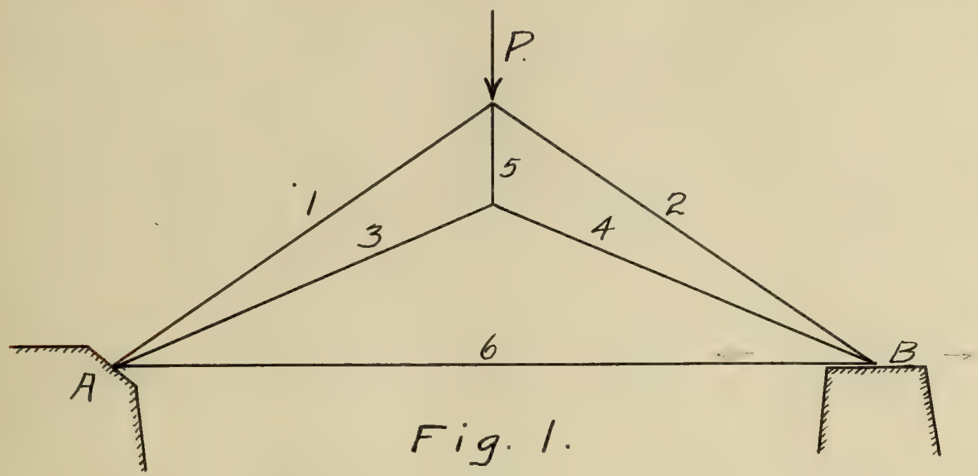


Fig. 1.

Let Fig. 1 represent a redundant system and let 6 be the redundant member.

The movement of the joint B will be considered. Since the members 1 to 5 constitute a rigid framework the movement at B is

$$\Delta = \sum_1^5 \frac{SuL}{AE}.$$

It is also evidently equal to the elongation of member 6 which is

$$\Delta = \frac{S_6 L_6}{A_6 E}.$$

The stresses in the members 1-5 may be considered as made up of two parts, namely:

(1) The stress S' due to the given loads considering the redundant member removed, and,

(2) The stress S'' due to a horizontal force, equal to the stress in 6 acting at B.

Since $\quad S'' = -S_6 u,$

Therefore $\quad \Delta = \sum_1^5 \frac{S'uL}{AE} - S_6 \sum_1^5 \frac{u^2 L}{AE}.$

$$\Delta = \frac{S_6 L_6}{A_6 E}.$$

$$S_6 = \frac{\sum_1^5 \frac{S'U L}{A E}}{\frac{L_6}{A_6 E} + \sum_1^5 \frac{U^2 L}{A E}}.$$

Evidently this result is true for a system with any number of members; and thus if r denotes the redundant member and n the number of necessary members we get the following general equation,

$$S_r = \frac{\sum_1^n \frac{S'U L}{A E}}{\frac{L_r}{A_r E} + \sum_1^n \frac{U^2 L}{A E}}.$$

Since in the general design of steel framework E is taken as constant for all members the equation may be written,

$$S_r = \frac{\sum_1^n \frac{S'U L}{A}}{\frac{L_r}{A_r} + \sum_1^n \frac{U^2 L}{A}}.$$

III. Practice.

1. Two Hinged Arch.

If the area of member 6 of the previous example be taken as infinite we have the condition of the two-hinged arch in which

$$S_6 = H.$$

Thus the horizontal thrust is given by the equation,

$$H = \frac{\sum_1^n \frac{S'UL}{A}}{\sum_1^n \frac{U^2L}{A}}.$$

If this equation were written out in expanded form it would be,

$$H = \frac{\frac{S_1'U_1L_1}{A_1} + \frac{S_2'U_2L_2}{A_2} + \cdots + \frac{S_n'U_nL_n}{A_n}}{\frac{U_2^2L_1}{A_1} + \frac{U_2^2L_2}{A_2} + \cdots + \frac{U_n^2L_n}{A_n}}.$$

If the members are properly designed,

$$\frac{S_1'}{A_1} = P, \qquad \frac{S_2'}{A_2} = P \cdots \text{etc.},$$

where P is the allowable stress.

Making this substitution the equation becomes,

$$H = \frac{PU_1L_1 + PU_2L_2 + \cdots + PU_nL_n}{\frac{PU_1^2L_1}{S_1'} + \frac{PU_2^2L_2}{S_2'} + \cdots + \frac{PU_n^2L_n}{S_n'}},$$

$$= \frac{P_t \sum_1^{n_t} UL + P_c \sum_1^{n_c} UL}{P_t \sum_1^{n_t} \frac{U^2L}{S'} + P_c \sum_1^{n_c} \frac{U_2L}{S'}},$$

in which P_t is the allowable tensile stress and P_c the compressive stress.

This equation offers no difficulties for the

tension members since P_t is usually taken as constant for any one structure, but the compression members offer some trouble since P_c varies with $\frac{l}{r}$. The formula for the value of P_c for live loads is,

$$P_c = 10000 - 45\,\frac{l}{r}.$$

In this equation $\frac{l}{r}$ varies from 40 to 100 and hence P_c varies from 5500 to 8200. For approximate results P_c may be taken as 7000 while P_t is usually 10000. If these values of P_c and P_t be used the equation becomes, since P_c is negative,

$$H = \frac{\sum_1^{n_t} UL - 0.7\sum_1^{n_c} UL}{\sum_1^{n_t} \frac{U^2 L}{S'} - 0.7\sum_1^{n_c} \frac{U^2 L}{S'}}.$$

The application of this equation will now be made by the determination of H for a two-hinged arch and comparing this result with the real value of H as determined after the members had been designed.

Example 1. — Let it be required to calculate the horizontal thrust for the Two-Hinged arch shown in Fig. 2.

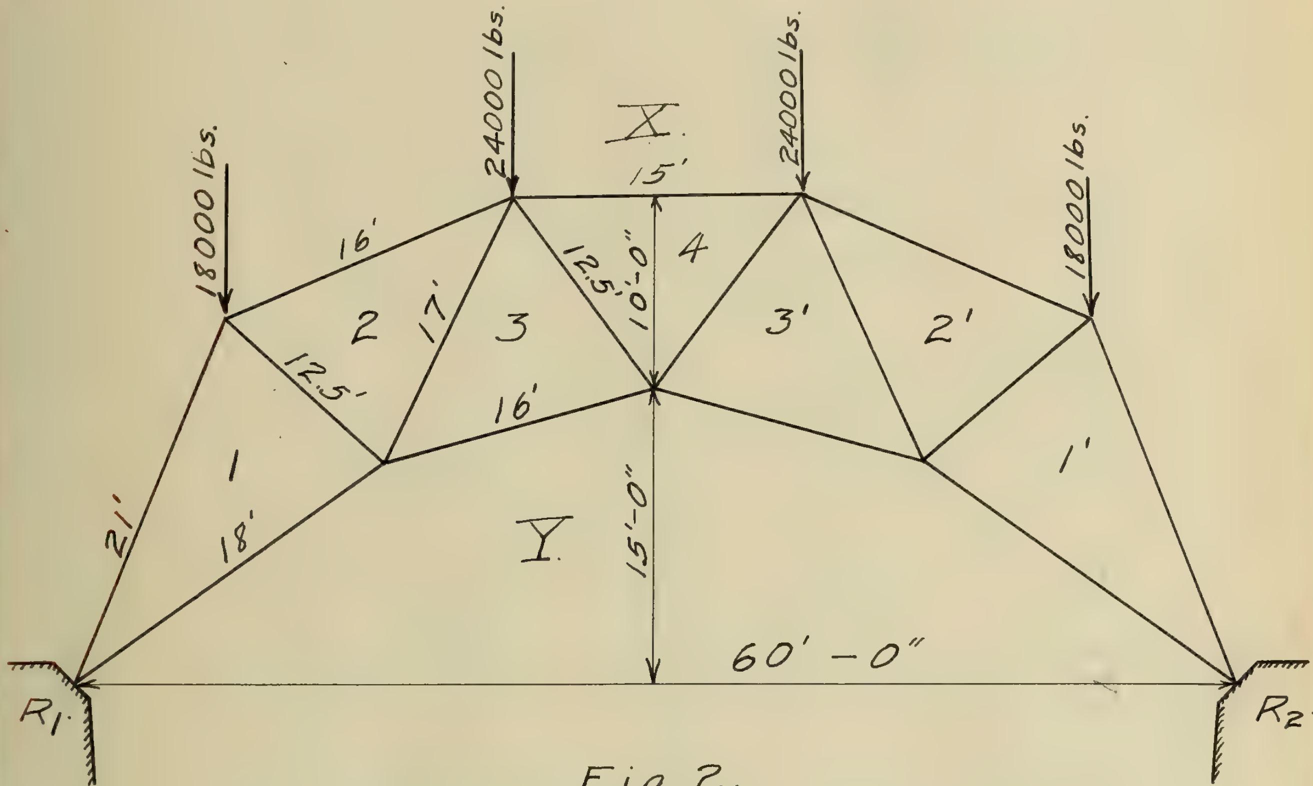


Fig. 2.

The data and results are given in Table I.

Table I.

Data and Results for Example I.

Member	Length in inches	S'	U	$(UL)n_t$ or $(-.7UL)n_c$	$(U^2L/S')n_t$ or $(-.7U^2L/S')n_c$
1-X	252	+60.0	−0.90	−226.8	+3.40
2-X	192	+41.0	−0.80	−153.6	+3.00
4-X	180	+67.0	−1.45	−260.0	+5.60
2′-X	192	+41.0	−0.80	−153.6	+2.98
1′-X	252	+60.0	−0.90	−226.8	+3.40
1-Y	216	−25.0	+1.60	−242.0	+15.50
3-Y	192	−57.0	+2.05	−274.0	+10.02
3′-Y	192	−57.0	+2.05	−274.0	+10.02
1′-Y	216	−25.0	+1.60	−242.0	+15.50
1-2	150	−30.0	+0.75	−78.6	+1.96
2-3	204	+32.0	−0.45	−91.0	+1.28
3-4	150	−22.0	+0.80	−84.0	+3.06
3′-4	150	−22.0	+0.80	−84.0	+3.06
2′-3′	204	+32.0	−0.45	−91.0	+1.28
1′-2′	150	−30.0	+0.75	−78.0	+1.96

From which there results,

$$\Sigma UL = -2559.4$$

$$\Sigma \frac{U^2L}{S} = +82.02$$

$$H = \frac{-2559.4}{82.02}$$

$$= -31,200 \text{ lbs.}$$

The value of H, as computed after the members had been designed, is given in the following table.

Table II.

Member	Area in Sq.in.	Length in ins.	S'	U	$\frac{SUL}{AE}$	$\frac{U^2L}{AE}$
1-X	5.3	252	+60.0	-0.90	-0.86	+.000,001,25
2-X	"	192	+41.0	-0.80	-.040	+.000,000,75
4-X	"	180	+67.0	-1.45	-.110	+.000,002,40
2'-X	"	192	+41.0	-0.80	-.040	+.000 000 75
1'-X	"	252	+60.0	-0.90	-.086	+.000,001,25
1-Y	"	216	-25.0	+1.60	-.054	+.000,003,45
3-Y	"	192	-57.0	+2.05	-.141	+.000,005,10
3'-Y	"	192	-57.0	+2.05	-.141	+.000,005,10
1'-Y	"	216	-25.0	+1.60	-.054	+.000,003,45
1-2	2.0	150	-30.0	+0.75	-.056	+.000,001,45
2-3	4.0	204	+32.0	-0.45	-.024	+.000,000,35
3-4	"	150	-22.0	+0.80	-.022	+.000,000,80
3'-4	"	150	-22.0	+0.80	-.022	+.000,000,80
2'-3'	"	204	+32.0	-0.45	-.024	+.000,000,35
1'-2'	2.0	150	-30.0	+0.75	-.056	+.000,001,45

$$\sum \frac{SUL}{AE} = -0.956$$

$$\sum \frac{U^2L}{AE} = +0.0000287$$

$$H = \frac{\sum \frac{SUL}{AE}}{\sum \frac{U^2L}{AE}} = \frac{-.956}{.0000287}$$

$$= -33400 \text{ lbs.}$$

A comparison of the two values of H shows that the first method gives a result in error by about $6\frac{1}{2}$ percent in this case. Since the value of H is dependent upon the cross-section areas of the members, it is readily seen that any formula, which gives the value of H before the arch has been designed, must necessarily be only approximate.

An examination of the formula,

$$H = \frac{\Sigma \frac{SuL}{A}}{\Sigma \frac{u^2 L}{A}}$$

shows that it is not the actual value of A that affects the result but the proportionate values for the different members. Thus, if A is the cross-section area of any member and αA, βA, etc. the areas of the remaining members, the equation may be written,

$$H = \frac{\frac{S_1 u_1 L_1}{A} + \frac{S_2 u_2 L_2}{\alpha A} + \frac{S_3 u_3 L_3}{\beta A} + \cdots}{\frac{u_1^2 L_1}{A} + \frac{u_2^2 L_2}{\alpha A} + \frac{u_3^2 L_3}{\beta A} + \cdots}$$

in which A will readily cancel out.

It was upon this principle that the above equation for finding H without knowing A was established, the relative ratios of A being obtained from the values of the stresses S in the members.

This is not strictly correct since a variation in the value of W for the different members will cause the ratios of the final stresses S to be somewhat changed from the ratios of the stresses S'.

Another consideration which effects the value of H, is the fact that the members are not always of the exact area as required by the design. There is a minimum size which is permitted which may often be large enough to carry two or three times the actual stress to which the member is subjected. Also, it is sometimes convenient to use the same cross-section for several members, as for instance in the upper or lower chords. In this case it is clearly the largest stress that governs the design and if this stress were used for all these members the approximation would be more nearly correct. If a limit were placed on the minimum size of angle to be used and a minimum value of S were taken to correspond to this value of the cross-section, the equation would give results more nearly correct.

In general, the formula is not intended to give an accurate value of H, but only an approximate value by means of which the members may be designed with a reasonable degree of accuracy, it being necessary in all cases to recalculate H and if this new value varies from the former value by any considerable amount, it will be necessary to recalculate the stresses and re-design the members.

2. Cross-Frames of Plate Girders.

A riveted cross-frame of a plate girder is a redundant system containing one redundant member. In actual practice it is customary to solve the stresses in the members by the ordinary methods of statics by neglecting one of the diagonal members, and then assuming that each diagonal takes one-half of the stress. The accuracy of this method will now be investigated by the solution of the stresses in some cross-frames as actually built, the stresses being determined by the method of least work.

Example 2.— Given the cross-frame shown in Figure 3, it is required to determine the stresses in the members.

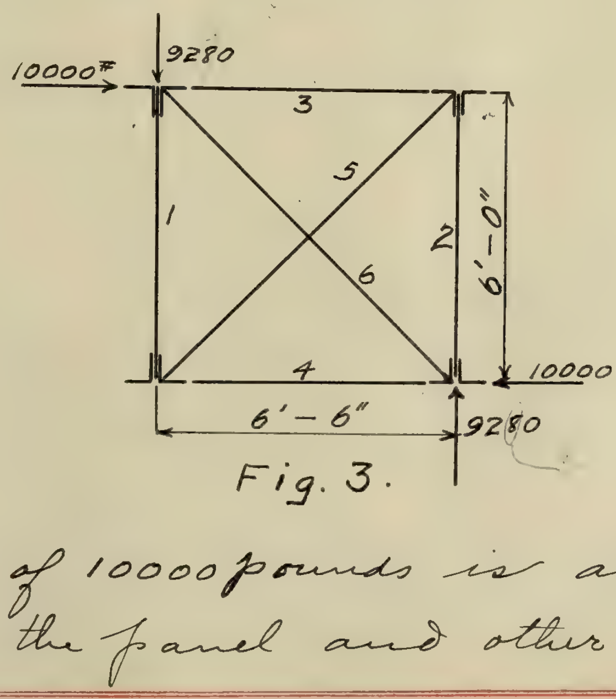


Fig. 3.

A shear of 10000 pounds is assumed to exist in the panel and other forces are

introduced to give equillibrium. The remaining data is given in Table 3. Member 6 is assumed to be the redundant member. The crossections of members 1 and 2 are so large that they are considered as rigid when compared to the other members. The values of the stresses are given by the equation,

$$S = S' + S_6 U.$$

where S' is the stress with the redundant member removed and S_6 is the stress in member 6. The value of S_6 is given by the equation,

$$S_6 = \frac{\sum_1^5 \frac{S'UL}{A}}{\frac{L_6}{A_6} + \sum_1^5 \frac{U^2L}{A}} .$$

Table III.

Data and Results of Example 2.

Member	L	A	S'	U	$\frac{UL}{A}$	$\frac{S'UL}{A}$	$\frac{U^2L}{A}$	S_6U	S
1	72	∞	+9.2	-0.678	0	0	0	-6.2	+3.0
2	72	∞	+9.2	-0.678	0	0	0	-6.2	+3.0
3	78	2	+10.0	-0.735	-28.7	-287.0	+21.2	-6.7	+3.3
4	78	2	+10.0	-0.735	-28.7	-287.0	+21.2	-6.7	+3.3
5	106	3	-13.6	+1.0	+35.3	-480.0	+35.3	+9.2	-4.4
6	106	3							
						-1054	+77.7		

$\frac{L_6}{A_6} = 35.3$ $\qquad$ $S_6 = -\frac{-1054}{35.3+77.7} = +9.2.$

If now, instead of considering the cross-section areas of members 1 and 2 as infinite, each be considered, in the case

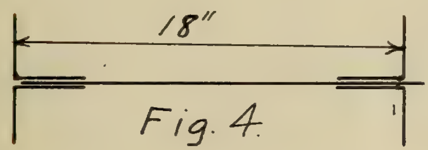


Fig. 4.

of end frames as composed of the four stiffener angles and the included web, as shown in Figure 4, the following results are obtained. Consider the angles as being $3\frac{1}{2}'' \times 3\frac{1}{2}'' \times \frac{3}{8}''$, spaced 18 inches back to back, and the web as $\frac{3}{8}''$ thick. Then,

$$A_1 = 2.49 \times 4 + \frac{3}{8} \times 18 = 16.7^{\square''}$$

and the results are as given in Table IV.

Table IV

Member	L	A	S'	U	$\frac{UL}{A}$	$\frac{S'UL}{A}$	$\frac{U^2L}{A}$	S_6U	S
1	72	16.7	+9.2	−0.678	−2.93	−26.9	+1.98	−6.4	+2.8
2	72	16.7	+9.2	−0.678	−2.93	−26.9	+1.98	−6.4	+2.8
3	78	2	+10.0	−0.735	−28.7	−287.0	+21.1	−7.0	+3.0
4	78	2	+10.0	−0.735	−28.7	−287.0	+21.1	−7.0	+3.0
5	106	3	−13.6	+1.0	+35.3	−480.0	+35.3	+9.5	−4.1
6	106	3							+9.5
						−1107.8	+81.46		

Hence,
$$\frac{L_6}{A_6} = \frac{106}{3} = 35.3$$

and
$$S_6 = -\frac{-1107.8}{35.3 + 81.46} = +9.5$$

Example 3. — In Figure 5 is given the dimensions of a standard end cross-frame used by the Northern Pacific for a 100-foot deck plate girder. Assuming a wind load on the upper chord of 600-pounds per linear foot, it is required to calculate the stresses in the members of the cross-frame.

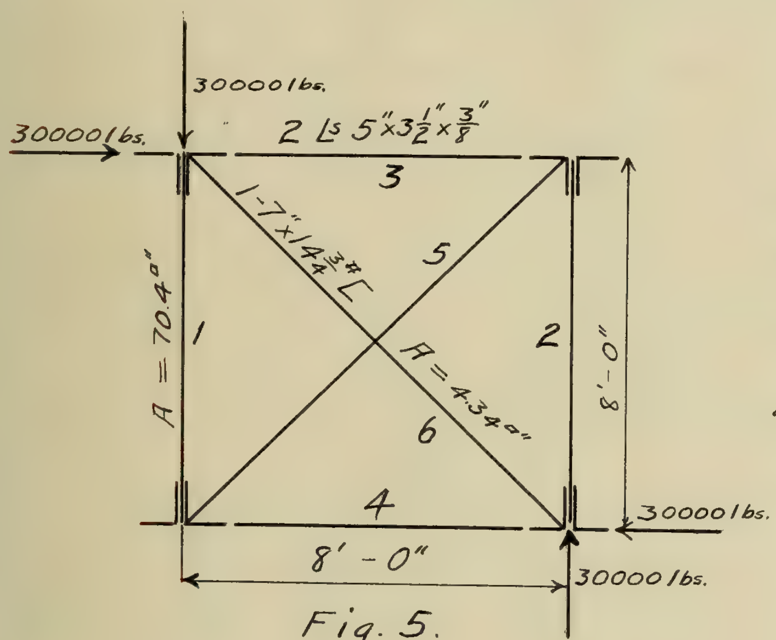


Fig. 5.

The shear in the end cross-frame

$= 600 \times 50 = 30000$ lbs.

The areas of members 1 and 2 are considered as made up of the stiffener angles and included web. Member 6 is taken as the redundant member. The data and results of this problem are given in Table V.

Table V.

Data and Results for Example 3.

Member	L.	A	S'	U	$\frac{UL}{A}$	$\frac{S'UL}{A}$	$\frac{U^2L}{A}$	S_6U	S.
1	96	70.4	-30.0	-0.707	-0.963	+28.9	+0.68	+18.0	-12.0
2	96	70.4	-30.0	-0.707	-0.963	+28.9	+0.68	+18.0	-12.0
3	96	6.1	-30.0	-0.707	-11.05	+331.5	+7.88	+18.0	-12.0
4	96	6.1	-30.0	-0.707	-11.05	+331.5	+7.88	+18.0	-12.0
5	135.7	4.34	+42.4	+1.0	+31.20	+1310.0	+31.20	-25.5	+16.9
6	135.7	4.34							-25.5
						+2030.8	+48.32		

From the table there results the following,

$$\frac{L_6}{A_6} = \frac{135.7}{4.34} = 31.2$$

and,

$$S_6 = -\frac{2030.8}{31.2+48.32} = -25.5\ K.$$

In Example 3 it is seen that members 3 and 4, although bearing less stress than the diagonals, have a much greater cross-section area. An example will now be given in which the diagonals have the greater cross-section area.

Example 4.— Let it be required to determine the stresses in the riveted cross-frame shown in Figure 6, due to a horizontal shear of 22000 pounds in the panel.

The cross section areas of members 1 and 2 are considered as made up of the

four stiffener angles and the included web. The arrangement is as shown

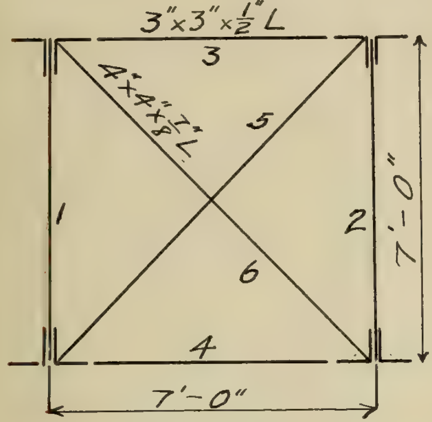


Fig. 6.

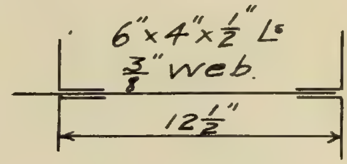


Fig. 7.

in Figure 7 and the area is:

$$A = 4.75 \times 4 + \tfrac{3}{8} \times 12\tfrac{1}{2} = 23.6 \text{ sq.in.}$$

Member 6 is taken as the redundant member. The results of the computations are for convenience arranged in tabular form, and are given in Table VI.

Table VI.

Member.	L	A	S'	u	$\frac{uL}{A}$	$\frac{S'uL}{A}$	$\frac{u^2L}{A}$	S_6u	S
1	84	23.6	-22.0	-0.707	-2.5	+55.0	+1.8	+16.4	-5.6
2	84	23.6	-22.0	-0.707	-2.5	+55.0	+1.8	+16.4	-5.6
3	84	2.75	-22.0	-0.707	-21.8	+482.0	+15.2	+16.4	-5.6
4	84	2.75	-22.0	-0.707	-21.8	+482.0	+15.2	+16.4	-5.6
5	119	6.24	+31.1	+1.0	+19.0	+592.0	+19.0	-23.2	+7.9
6	119	6.24							-23.2
						+1666.0	+53.0		

There results the following,

$$\frac{L_6}{A_6} = \frac{119}{6.24} = +19.0 \quad \text{and,} \quad S_6 = -\frac{+1666}{19+53} = -23.2 \text{ K.}$$

3. Diagonals of Riveted Trusses.

In a riveted truss a panel having two diagonals forms a redundant system with one redundant member. In this case as in the case of the cross frames of plate girders it is customary to design each diagonal to take one-half the shear in the panel. By the method of least work the actual stresses in the diagonals of some trusses will now be determined.

Example 5. — In Figure 8 is shown the center panel of a standard single-track five panel riveted truss as built by the American Bridge Company. It is required to determine the stresses in the diagonals.

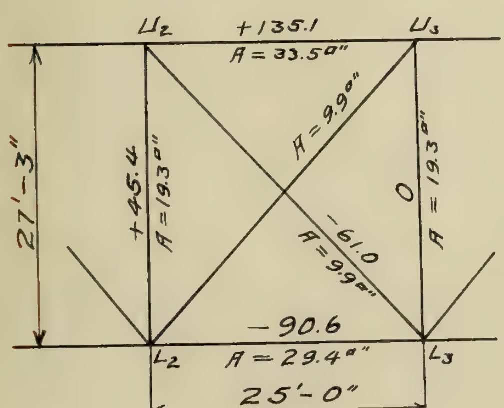


Fig. 8.

The stresses given in the figure are determined by loading with Coopers E-50 loading so as to produce the maximum shear in the panel, and considering L_2U_3 as the redundant member.

The results are given in Table VII.

Table VII.

Data and Results for Example 5.

Member.	L	A	S'	U	$\frac{UL}{A}$	$\frac{S'UL}{A}$	$\frac{U^2L}{A}$	S_rU	$S.$
U_2U_3	300	33.5	+135.1	−0.672	−6.0	−810.6	+4.0	−19.8	+115.3
L_2L_3	300	29.4	−90.6	−0.672	−6.9	+630.0	+4.7	−19.8	−110.4
U_2L_2	327	19.3	+45.4	−0.732	−12.4	−565.0	+9.1	−21.5	+19.9
U_3L_3	327	19.3	0	−0.732	−12.4	0	+9.1	−21.5	−21.5
U_2L_3	444	9.9	−61.0	+1.0	+44.8	−2732.0	+44.8	+29.4	−31.6
U_3L_2	444	9.9		−1.0					+29.4
						−3477.6	+71.7		

$$\frac{L_r}{A_r} = \frac{444}{9.9} = +44.8$$

$$S_r = -\frac{-3477.6}{44.8 + 71.7} = +29.4$$

These results show that for this particular bridge the method of assuming each diagonal to take one-half of the stress is not far from correct. It should also be noted that, while the stresses in most of the members are decreased, the stress in L_2L_3 is increased by over 20 percent. It is not necessary to consider this since the maximum stresses in the chord members occur under entirely different loading; a loading such that the shear in the panel is almost zero.

Example 6.— Compute the stresses in the diagonals of the riveted truss shown in Fig. 9.

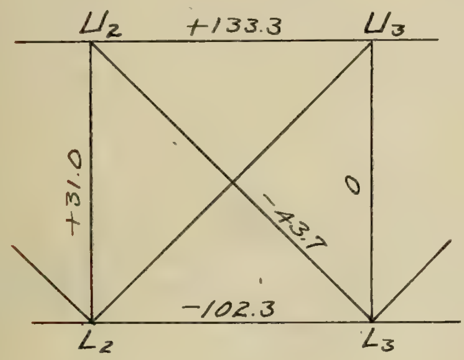


Fig. 9.

U_3L_2 is taken as the redundant member. The data and results are given in Table VIII.

Table VIII.

Data and Results for Example 6.

Member.	A	L	S'	U	$\frac{UL}{A}$	$\frac{S'UL}{A}$	$\frac{U^2L}{A}$	S_rU	S.
U_2U_3	22.2	240	+133.3	−0.71	−7.7	−1022.0	+5.42	−12.1	+121.2
L_2L_3	10.2	240	−102.3	−0.71	−16.7	+1704.0	+11.82	−12.1	−114.4
U_2L_2	5.2	240	+31.0	−0.71	−33.0	−1022.0	+23.42	−12.1	+18.9
U_3L_3	5.2	240	0	−0.71	−33.0	0	+23.42	−12.1	−12.1
U_2L_3	4.4	339	−43.7	+1.0	+72.5	−3390.0	+77.50	+17.0	−26.7
U_3L_2	4.4	339		−1.0					+17.0
						−3730	+141.6		

$$\frac{L_r}{A_r} = \frac{339}{4.4} = 77.5$$

$$S_r = -\frac{-3730}{77.5+141.6} = +17.0\,K.$$

Thus the stress in U_2L_3 is seen to be −26700 lbs, whereas if it is designed to take one-half the shear it would be designed for a stress of 21800 lbs., a value which is over 20 percent too small.

From the previous examples it is seen that the method of assuming each diagonal to take one-half of the stress in the case of cross-frames and riveted trusses can not always be relied upon to give correct results, or results on the side of safety. In fact this assumption will give correct results only under the most favorable cases and any variation from the assumed value always gives a result larger than the assumed value in one of the diagonals.

In the case of plate girder cross-frames the variation was found in Example 2 to be as much as 50 percent over what it would be if each diagonal took one-half of the shear in the panel.

The cases investigated also seem to show that the larger the cross-section areas of the diagonals in comparison with the cross-section areas of the other members, the greater the variation in the stresses in the diagonals. The results obtained for the diagonals of riveted trusses do not vary so much from the assumed values. In Example 6, however, the actual value of

the stress in U_2L_3 is seen to vary from the assumed value by over 20 percent, an amount worthy of note in the design of members. The variation is not so great in the diagonals of riveted trusses as it is in the diagonals of cross-frames of plate girders, because in riveted trusses the ratios of the areas of the diagonal cross-sections to the cross-section areas of the other members is less than it is in plate girder cross frames.

After the members have been designed, the stresses in the diagonals can be easily computed by the method of least work and thus the actual efficiency of the members can be determined and in case this is less than unity, the members should be re-designed.

4. Stresses in Steel Towers.

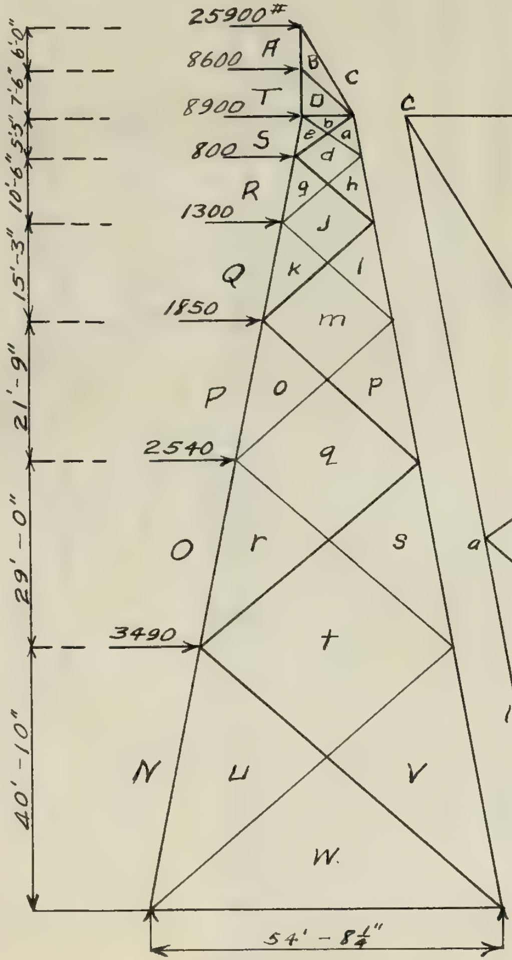
The transverse and also the longitudinal bracing of trestle towers is composed of rigid diagonals capable of taking either tension or compression, and hence redundant systems are formed. The ordinary assumption that one-half the shear in the panel is taken by each diagonal is usually made in the design of these members.

The accuracy of this method will now be investigated by the application of the method of least work for determining the stresses in the members.

Example 7.— Let it be required to determine the stresses in one of the towers of the Richland Creek Viaduct due to wind when a train is on the tower.

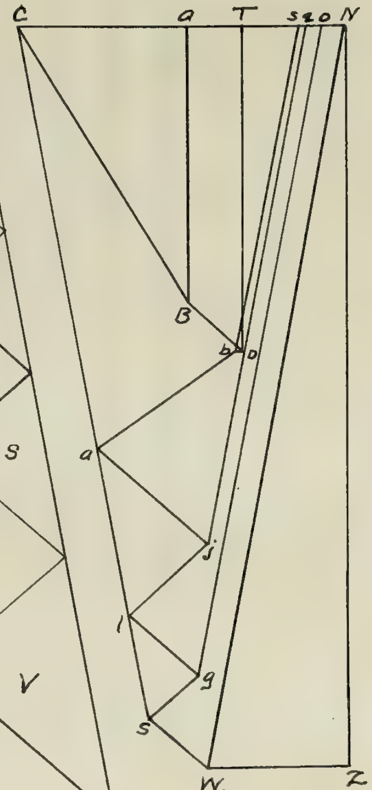
The wind load is taken as given by Cooper in his specifications, and the forces are considered as acting as indicated on plate I. The values of S' are determined graphically on plate I and the cross-section areas of the members together with the results of the computations are all given in Table IX.

PLATE I.



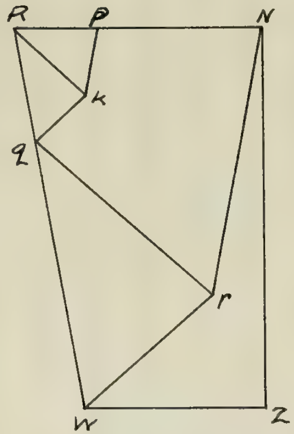
Scale 1 in. = 30 ft.

Fig. 1.



Scale 1 in. = 30000 lbs.

Fig. 2.



Scale 1 in. = 3000 lbs.

Fig. 3.

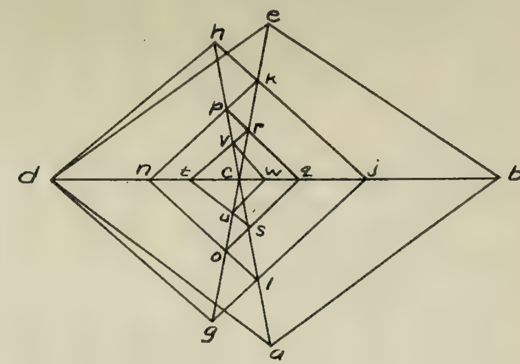


DIAGRAM FOR 1 LB. COMPRESSION IN REDUNDANT MEMBER 'be'.

Fig. 4.

WIND STRESS DIAGRAMS FOR TOWER OF

RICHLAND CREEK VIADUCT

FOR DETERMINING S'.

be = Redundant Member.

Table IX.

Data and Results for Example 7.

Member.	L	A	S'	u	$\frac{uL}{A}$	$\frac{S'uL}{A}$	$\frac{u^2L}{A}$	$S_r u$	S.
ca	68	37.72	+66000	-0.55	-1.0	-66000	+0.549	-6800	+59200
bc	96	11.76	+1000	-0.95	-7.75	-8000	+7.35	-11800	-10800
be	128	11.76							
ec	68	37.72	-54000	-0.55	-1.0	+54000	+0.549	-6800	-60800
ed	128	11.76	-24000	+1.0	+10.90	-264000	+10.90	+12400	-11600
cg	127	37.72	-54000	+0.55	+1.85	-100000	+1.02	+6800	-47200
gd	192	11.76	+22000	-0.90	-14.70	-310000	+13.20	-11600	+10400
dh	192	11.76	0	-0.90	-14.70	0	+13.20	-11600	-11600
hc.	127	37.72	+66000	+0.60	+2.02	+132000	+1.22	+7400	+73400
ck	184	37.72	-82000	-0.42	-2.25	+184000	+0.94	-5200	-87200
kj	270	11.76	+2000	+0.63	+14.45	+29000	+9.10	+7800	+9800
jl	270	11.76	-16000	+0.63	+14.45	-230000	+9.10	+7800	-8200
lc	184	37.72	+92000	-0.42	-2.25	-210000	+0.94	-5200	+86800
co	264	37.72	-82000	+0.30	+2.10	-172000	+0.63	+3700	-78300
on	384	11.76	+14000	-0.44	-14.35	-200000	+6.30	-5500	+8500
np	384	11.76	-1000	-0.44	-14.35	+14000	+6.30	-5500	-6500
pc	264	37.72	+93000	+0.30	+2.10	+195000	+0.63	+3700	+96700
cr	353	44.64	-102000	-0.20	-1.58	+160000	+0.31	-2500	-104500
rq	528	11.76	+4000	+0.30	+13.50	+54000	+4.05	+3700	+7700
qs	528	11.76	-10000	+0.30	+13.50	-135000	+4.05	+3700	-6300
sc	353	44.64	+109000	-0.20	-1.58	-170000	+0.31	-2500	+106500
cu	418	44.64	-119000	+0.17	+1.59	-190000	+0.27	+2100	-116900
ut	744	11.76	+12000	-0.23	-14.50	-174000	+3.34	-2900	+9100
tv	744	11.76	-3000	-0.23	-14.50	+42000	+3.34	-2900	-5900
vc	418	44.64	+114000	+0.17	+1.59	+180000	+0.27	+2100	+116100
wz	656	11.76	-24000	+0.13	+7.25	-175000	+0.93	+1600	-22400
						-1360000	+98.80		

$$\frac{L_r}{A_r} = \frac{128}{11.76} = 10.9$$

$$S_r = -\frac{-1,360000}{10.9 + 98.80} = +12400 \text{ lbs.}$$

Example 8.— It is required to find the stresses in the longitudinal bracing of a trestle tower due to the braking of a train on the tower.

The towers are considered as 30 feet wide and 60 feet apart, the loading is taken as Coopers E-40 and the coefficient of friction between the wheels and rails is assumed to be 0.2. By placing wheel 2 at A the maximum value of the load W is obtained, and is found to be 235000 lbs.

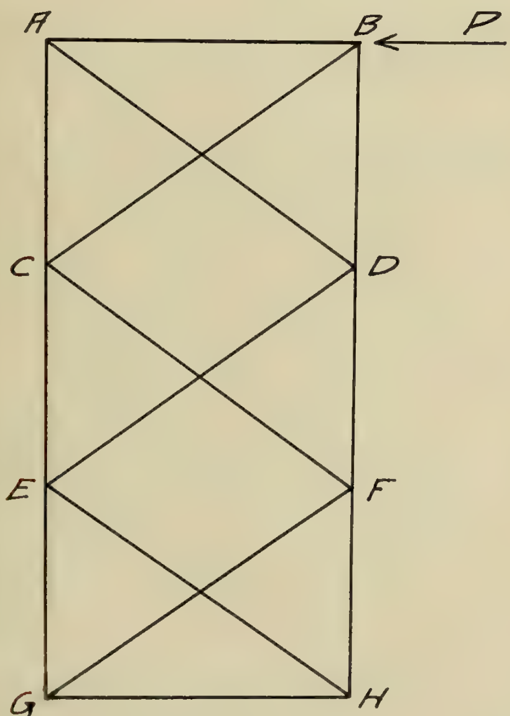
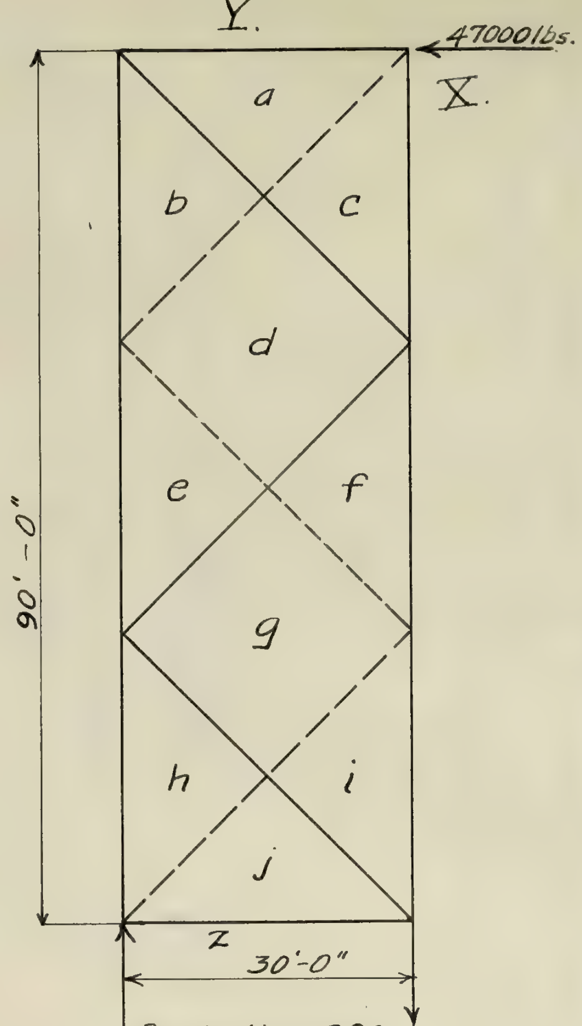


Fig. 10.

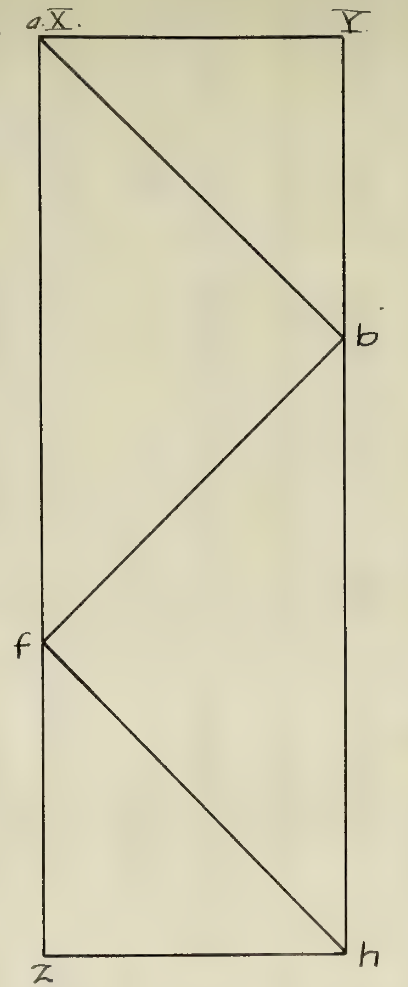
$$P = 0.2\,W$$
$$= 0.2 \times 235000$$
$$= 47000 \text{ lbs.}$$

The values of S' are determined graphically on Plate II, the member BC being considered as redundant. The lengths and cross-section areas of the members and also the results of the computations are given in Table X.

PLATE II.



Scale 1 in. = 20 ft.
Fig. 5.



Scale 1 in. = 30000 lbs.
Fig. 6

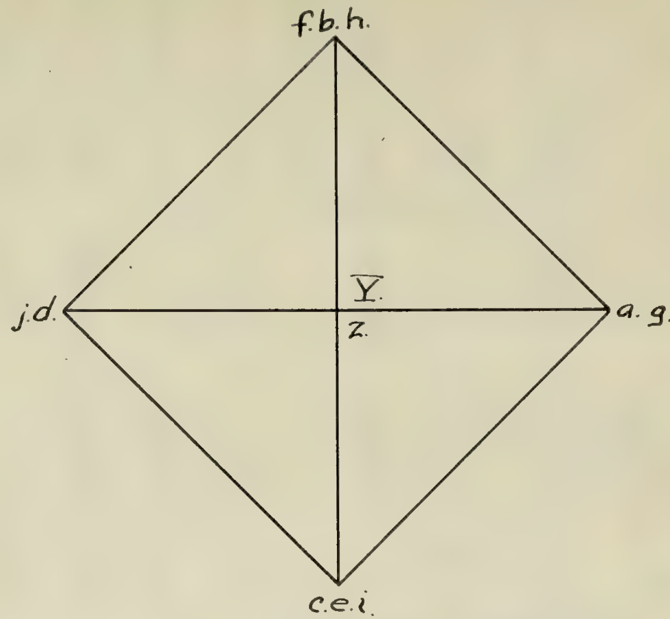


DIAGRAM FOR 1 LB. COMPRESSION IN REDUNDANT MEMBER 'ac'.

Fig. 7.

DIAGRAM FOR STRESSES IN TOWER DUE TO BRAKING OF TRAIN.

Table X.

Data and Results for Example 8.

Member.	L	A	S'	U	$\frac{UL}{A}$	$\frac{S'UL}{A}$	$\frac{U^2L}{A}$	S_rU	$S.$
ya	360	37.72	+47000	-0.707	-6.7	-315000	+4.7	-23700	+23300
ac	509	5.0							+33600
ab	509	5.0	-66000	+1.0	+101.8	-6,750,000	+101.8	+33600	+32400
cx	360	37.72	0	-0.707	-6.7	0	+4.7	-23700	-23700
by	360	37.72	+47000	-0.707	-6.7	-315000	+4.7	-23700	+23300
bf	509	5.0	+66000	-1.0	-101.8	-6,750,000	+101.8	-33600	+32400
de	509	5.0	0	-1.0	-101.8	0	+101.8	-33600	-33600
fh-g	509	5.0	-66000	+1.0	+101.8	-6,750,000	+101.8	+33600	-32400
ey	360	37.72	+47000	+0.707	+6.7	+315000	+4.7	+23700	+70700
fx	360	37.72	-94000	+0.707	+6.7	-630000	+4.7	+23700	-70300
gi	509	5.0	0	+1.0	+101.8	0	+101.8	+33600	+33600
hy	360	37.72	+141000	-0.707	-6.7	-945000	+4.7	-23700	+116300
ix	360	37.72	-94000	-0.707	-6.7	+630000	+4.7	-23700	-117700
zj-h.	360	37.72	+47000	-0.707	-6.7	-315000	+4.7	-23700	+23300
						-21,826,000	+546.6		

$$\frac{L_r}{A_r} = \frac{509}{5} = 101.8$$

$$S_r = -\frac{-21,825,000}{101.8+546.6}$$

$$= +33\,600 \text{ lbs.}$$

In each of the above examples it is seen that the stress which each diagonal is required to take does not differ much from the value given for the stress by assuming that each diagonal takes one-half the shear in the panel. Thus in Example 7 this assumption gives the stress in the diagonal <u>be</u> as +12000 pounds, whereas the actual stress is seen to be +12400, an increase of only 3 per cent over the assumed value. Also in Example 8 the ordinary assumptions give a stress in the member <u>ac</u> equal to +33000 pounds while the method of least work gives the value +33600, an increase of less than 2 per cent.

The cause of this small variation is that the cross-section areas of the diagonal members are small in comparison to those of the main members. If, in Example 8, the cross-section of the diagonals be assumed equal to that of the main members the increase of the actual over the assumed value becomes about 8 per cent and by still further increasing the areas of the diagonals the increase of stress becomes still greater.

IV. CONCLUSION.

It is evident from the cases investigated that it is only under the most favorable circumstances that the method of assuming each diagonal in a riveted cross-frame or trestle tower to take one-half the shear in the panel, will give values that agree closely with the actual values, while, in many cases, the actual values may be 50 per cent or even more in excess of the assumed values.

It may be given as a general rule that the larger the cross-section area of the diagonals in comparison to that of the other members the greater the variation between the stresses in the diagonals, and as this area becomes relatively smaller in comparison to the other members the amount of variation in the stresses becomes smaller.

Thus in the case of plate-girder cross-frames where the diagonals were larger than the struts, the variation in the stresses of the diagonals was the greatest and in trestle-towers where the diagonals are small compared to the legs the variation in the diagonal stresses was least.